SUBMARINE PONTOON SUPPORTED VESSEL
WITH ACTIVE DISPLACEMENT CONTROL

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DEPARTMENT OF NAVAL ARCHITECTURE
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ABSTRACT

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Submarine Pontoon Supported Vessels are being used or considered for drilling platforms, oceanographic research ships, and mobile satellite tracking stations due to their inherently small motions. This thesis explores the possibility of reducing these motions still further by using active displacement control. The equations of motion for a typical model are derived and solved for heave, pitch, and roll. The forces required to reduce the motions are optimized and the resulting motions are determined. The power required to maintain a specified motion amplitude is derived.

The reduction of heave amplitude of a vessel near resonance requires excessive power, indicating that the vessel should be designed with a natural period above that of the expected waves. However, the use of active displacement control to provide essentially zero heave amplitude in a moderate sea state is potentially attractive due to the small average power requirement.

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NOTATION

A	Incident wave amplitude
b	Height of the SPSV's center of gravity above calm waterline
c	subscript applying to a column
C	Drag coefficient of a column
c_{am}	Added mass coefficient
c_d	Drag coefficient
d_c	Diameter of a column
d_h	Diameter of a hull
H	Wetted length of a column
h	Subscript applying to a hull
i	Subscript referring to a particular segment
I_x, I_y, I_z	Platform moments of inertia about x, y, z axes
k	Wave number = $2\ /L_w = w^2/g$
L	Length of a hull
L_i	Length of a hull segment = $L/2$
L_w	Wave length
M	SPSV mass
m_c	displaced column mass
m_h	displaced hull mass
Δm	Added mass or hydrodynamic inertia $=c_{am} m$
S	$\cos(kx_c \cos\alpha) \cos(ky_c \sin\alpha)$
t	time variable
T	period = $1/w$
X, Y, Z	Force components
x, y, z	Cartesian coordinate system
x_{ci}, y_{ci}	Location of axes of i-th coordinate system
z_h	vertical distance of a hull axis below the SPSV's center of gravity
α	Incident wave angle
δ	Damping parameter

ξ, η, ζ	Linear displacement variables
ϕ, θ, ψ	Angular displacement variables
ρ	Water density
σ	Wave profile
τ	$k(x \cos\alpha - y \sin\alpha) - wt$
τ_i	$k(x_{ci} \cos\alpha - y_{ci} \sin\alpha) - wt$
w	Wave frequency $= 1/T_w$
Λ	Amplitude factor = ratio of controlled to uncontrolled motion
Q_o	$\frac{1}{H} \int_H e^{k(z + b)}\, dz = \frac{1}{kH} (1-e^{-kH})$
Q_1	$\frac{1}{H} \int_H e^{k(z+b)} z\, dz = \frac{1}{k^2 H} \left[kHe^{-kH} - (1+kb)(1-e^{-kH}) \right]$
x_2^2	$\frac{1}{L_i} \int_{L_i} x^2\, dx = L^2/12$
$z_1 = z_c$	$\frac{1}{H} \int_H z\, dz = -(\frac{H}{2} + b)$
z_2^2	$\frac{1}{H} \int_H z^2\, dz = H^2/3 + Hb + b^2$
μ	$\dfrac{e^{-k(z_h - b)}}{\frac{kL}{H} \cos\alpha} \sin (\frac{kL}{4} \cos\alpha)$
μ_e	$\dfrac{e^{-k(z_h - b)}}{\frac{k^2 L}{2} \cos^2\alpha \cos(kx_c \cos\alpha)}$
K_c	$1 - \cos (\frac{kL}{2} \cos\alpha) - \frac{kL}{2} \cos\alpha \sin (\frac{kL}{2} \cos\alpha)$
K_s	$- \sin (\frac{kL}{2} \cos\alpha) + \frac{kL}{2} \cos\alpha \cos (\frac{kL}{2} \cos\alpha)$

I. INTRODUCTION

In recent years a considerable amount of attention has been focused on floating platforms for a wide variety of missions. These platforms have been identified by many names, including "Mobile Column Stabilized Platforms (MCSP)" and "Stable Ocean Platforms (SOP)". For purposes of this paper they will be referred to as "Submarine Pontoon Supported Vessels (SPSV)". The SPSV is characterized by its relatively small wave induced motions. It consists of a platform type structure supported by columns attached to submarine hulls. The main buoyancy elements are the hulls, and since they are located below the area of major wave effects, the motions of the vessel are small. The supporting columns, which are exposed to wave action, are widely spaced, small in relation to the wave lengths, and present the same minimum cross section and waterplane area to the sea in any orientation. As a result of these characteristics, the SPSV is useful for missions which require a ship to operate at a specific point in the ocean with minimum motions. Speed in transit may be important but remains secondary to the motion-free characteristics. Since the major working areas are not in direct contact with the surface of the sea, the design and utilization of the upper structure need not be restricted by hydrodynamic considerations, as is the case with a conventional surface ship. Thus unconventional arrangements and unique configurations are possible to fulfill specific tasks.

The oil industry has been the first to design, construct, and operate a SPSV. Deep water drilling requires a floating structure, and the SPSV has filled this need. The platform provides sufficient space for the drilling rig and associated equipment, and the small motion of the vessel permits drilling in low sea states

without problem. Most of the existing platforms are non-self propelled, and they are positioned at sea by anchoring systems. However, as the need for drilling in deeper water increases, the platforms will be equipped with dynamic positioning devices to maintain station.

The use of the SPSV for scientific research holds great potential. The platform provides considerable volume and deck area for specialized equipment and scientific facilities. Due to the considerably reduced motions as compared with a conventional ship, personnel comfort, and hence effectiveness, is high. As a further consequence of reduced motions, laboratory facilities can be more sophisticated, and personnel can continue to perform research in weather conditions which would prohibit work in a conventional ship. The MOHOLE vessel, although not actually constructed, underwent considerable design effort. The vessel was intended to drill through the earth's mantle in very deep water. The vessel consisted of a large rectangular platform supported on two submerged hulls by six cylindrical columns. The design called for a propulsion system capable of approximately 10 knots, and a positioning system using ducted thrusters in the hulls.

The National Aeronautics and Space Administration (NASA) has shown interest in a SPSV for use in missile and space vehicle tracking and communications. The requirements for ballistic missile and space missions are strict, and since these missions often require telemetry and tracking coverage over ocean areas, a sea-going tracking facility is necessary. Present range tracking ships are significantly limited in that the pitching and rolling of the ship create radar data inaccuracies for which adequate compensation cannot be provided. In order to minimize the effects of wind, current, and waves, present tracking ships must normally maintain headway, covering a path which further complicates the tracking problem. Crew discomfort during adverse weather, and lack of space for

recreation become significant problems during extended missions.

The SPSV overcomes these disadvantages and provides other unique advantages. Since the motions of the SPSV are considerably less than those of present ships, data accuracy is improved, equipment reliability is increased, and personnel discomfort is alleviated. The above water platform can be optimized to suit the mission, and radars and other electronic equipment can be located more effectively than on a conventional ship form. Additional space is also available for living areas and recreation. The SPSV would not be required to steam in a pattern because, since it is relatively insensitive to wind and wave direction, a dynamic positioning system would permit "hovering" over a specific spot. This capability implies that the endurance of the SPSV on station would be greater than that of a conventional range tracking ship.

The SPSV has potential as a support vessel for deep diving submersibles, either for oceanographic research or for submarine rescue. Small, deep diving submersibles have limited range and require close support. The SPSV provides a relatively motionless vessel which could house or support such submersibles.

There are a number of other possible missions for the SPSV, although some may be well in the future. The salvage potential is large due to the capability of lifting, large weights without developing large pitch or roll moments. Underwater structures, habitats, and arrays will require substantial tending, particularly during construction, and the SPSV has a unique capability to provide such support. Further uses such as a helicopter landing platform or a sea going hotel are conceivable.

With all of the foregoing missions, the ability of the SPSV to be located at a fixed point, with small motion, and yet be capable of moving from one location to another are primary considerations. In many cases it might be desirable

to minimize motions still further. This might be necessary to provide an essentially motionless platform, or to extend the working range of the SPSV into higher state seas. As the natural period of heave, pitch, and roll are large for the SPSV, it might be feasible to control motions with the use of an active displacement control system. The system envisioned would consist of additional ballast tanks and the pumping capacity to offset wave induced motions by alternately increasing and decreasing ballast. If the water were added and removed at the proper location and with the proper phase relation to the exciting wave forces, the motions of the SPSV in heave, roll, and pitch could theoretically be reduced to near zero.

This investigation considers the development of a active displacement control system for a SPSV. To determine the feasibility of such a system, the power requirements are determined for three situations. The first is the maintenance of essentially zero motions in low to moderate seas. The second is the significant reduction of heave in heavy seas. Finally the reduction of heave, at resonance, to a reasonable level is considered. The alteration of the SPSV's natural period is also discussed.

II. DESCRIPTION OF SPSV MODEL

The SPSV consists of two cylindrical hulls which are joined to the working platform by four cylindrical columns. The working platform is rectangular in shape. Figure 1 shows a simplified lines drawing of the SPSV.

A Cartesian coordinate system (x, y, z) is established with the origin located at the undisturbed center of gravity (KG). The x-axis is positive towards the bow, the y-axis is positive to port, and the z-axis is positive upwards. The undisturbed water surface is the plane $z = -b$.

It is assumed (ref. 1) that the columns are far enough apart so that each column and the portion of the hull directly beneath can be considered a hydrodynamically separate segment, and any interactions between segments can be neglected. The segments are numbered as shown in figure 2.

The hull axes are parallel to the x-axis and the column axes are parallel to the z-axis. For convenience, an additional coordinate system (x_i, y_i, z_i) is introduced in the i^{th} segment with the origin at $(x_{ci}, y_{ci}, 0)$ of the primary coordinates.

Displacements of the center of gravity are described by surge (ξ), sway (η), and heave (ζ), and rotations about the x, y, and z axes by roll (ϕ), pitch (θ), and yaw (ψ). Displacements of a point located on a column axis (x_{ci}, y_{ci}, z_i) can be related to displacements of the center of gravity by:

$$\xi_i = \xi + z_i \theta - y_{ci} \psi \qquad (1\text{-a})$$

$$\eta_i = \eta + x_{ci} \psi - z_i \phi \qquad (1\text{-b})$$

$$\zeta_i = \zeta + y_{ci} \phi - x_{ci} \theta \qquad (1\text{-c})$$

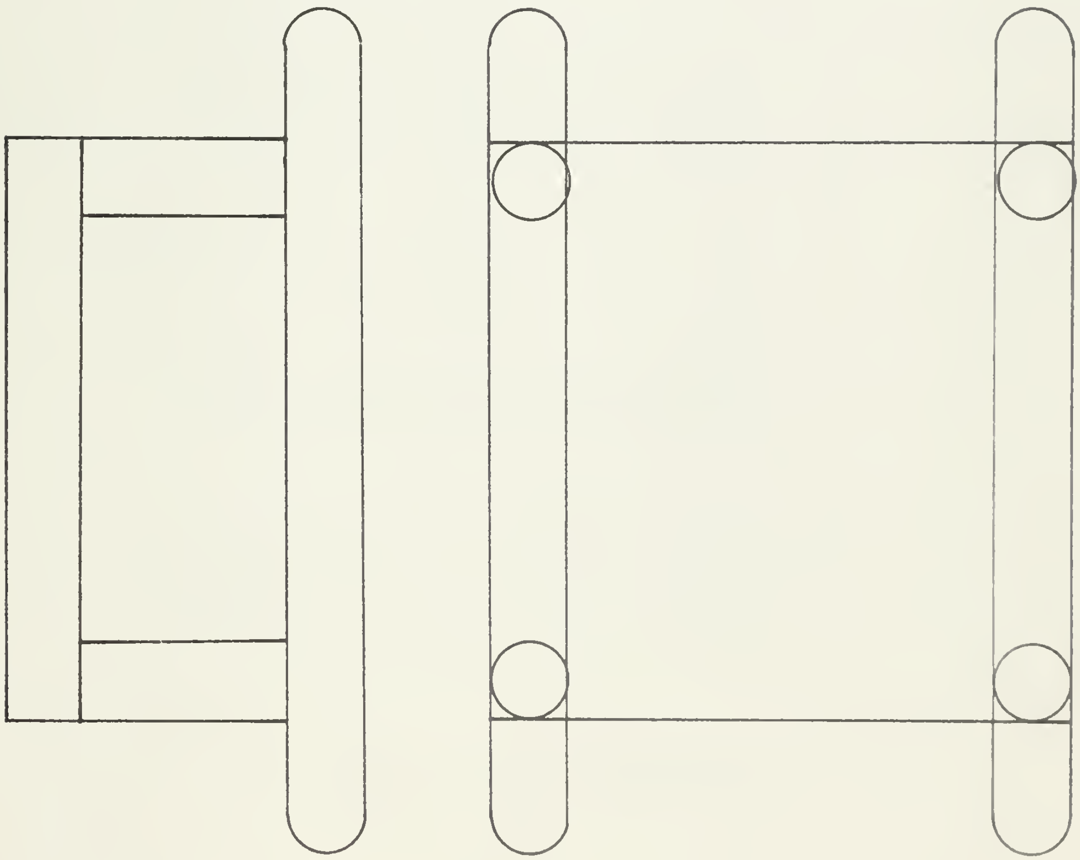

Figure 1 SPSV Lines

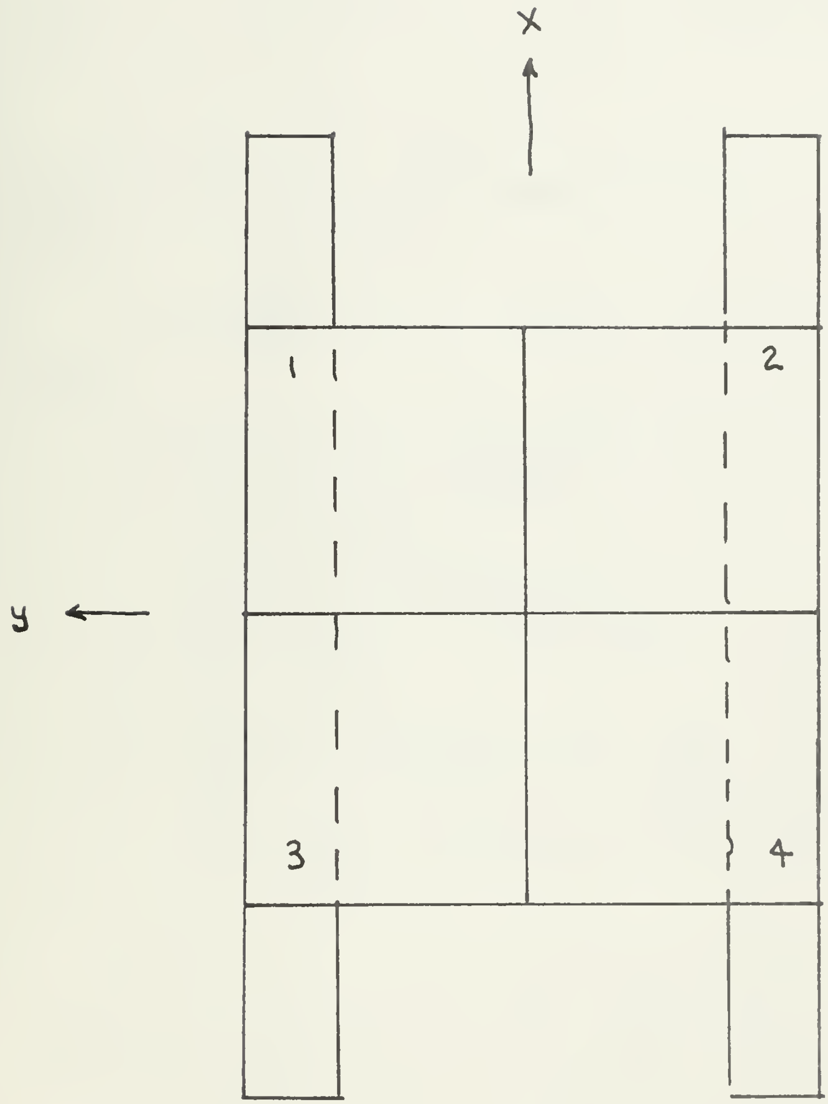

Figure 2 Segment Numbering

III. DERIVATION OF FORCES AND EQUATIONS OF MOTION

Wang (ref. 1) has derived the motions and forces of a floating platform. The derivation of the motions and forces of the SPSV draw heavily from this work.

It is assumed that the SPSV is subjected to a regular train of two dimensional deep water gravity waves whose velocity potential is of the form:

$$\Phi = -A\frac{w}{k}\, e^{k(z + b)} \sin \tau \qquad (2)$$

From the theory of gravity waves (ref. 3, 4), the wave profile can be written:

$$\sigma = \frac{1}{g}\frac{\partial \Phi}{\partial t} = Ae^{k(z + b)} \cos \tau \qquad (3)$$

where:

τ = k(x cos α - y sin α) -wt

g = gravitational constant

t = time variable

A = surface wave amplitude

k = wave number = $2\pi/L_w = w^2/g$

L_w = wave length

w = circular frequency of the wave

α = angle of the wave front measured clockwise from the negative x-axis. (For port beam seas, $\alpha = 90^\circ$, for head seas, $\alpha = 180^\circ$).

It is further assumed that the column and hull diameters are small compared to the wave length, therefore the motion of the water across the diameters can be considered uniform.

The forces acting on the SPSV are hydrostatic and hydrodynamic. The hydrostatic force is the restoring force expressed as:

$$\text{Hydrostatic Force} = \frac{\pi}{4}\rho g\, d_c^2 (A\cos\tau_i - \zeta_i) \qquad (4)$$

The hydrodynamic forces are caused by changes in pressure on the segment due to relative velocities and accelerations. Those hydrodynamic forces in phase with accelerations are termed intertial, and those in phase with velocities are termed damping.

The inertial forces consist of a wave force and a force due to the added mass of the segment. The wave force is due to the pressure on the segment resulting from the segment's interference with the acceleration of the water particles. Korvin - Kroukovsky (ref. 4, chapter 2) expresses this force as:

$$\text{Wave Force} = -(1 + k)\ (\text{pressure gradient})\ (\text{body volume}) \qquad (5)$$

From gravity wave theory, the pressure is:

$$p = \rho \frac{\partial \Phi}{\partial t} = \rho g \sigma \qquad (6)$$

Substituting equation (3) into equation (6), the expression for pressure becomes:

$$p = \rho g A e^{k(z + b)} \cos \tau \qquad (7)$$

The pressure gradient is easily calculated from equation (7).

In equation (5), k is the coefficient of accession to inertia, and 1 + k is defined in reference 4, chapter 2 as:

$$1 + k = \frac{\text{total inertia of a body floating in a fluid}}{\text{inertia of the fluid displaced by the body}} \qquad (8)$$

Equation (8) can also be written:

$1 + k = \dfrac{m + \Delta m}{m}$, where Δm is the added mass or hydrodynamic inertia.

The inertia force due to the hydrodynamic inertia is the product of the hydrodynamic inertia and the relative acceleration. The damping force is the product of the damping coefficient and the relative velocity between the water and the segment.

The water velocities in the x, y, and z directions can be obtained from the velocity potential, equation (2), as follows:

$$V_x = -\frac{\partial \Phi}{\partial x} = Awe^{k(z + b)}\cos\alpha \cos\tau \qquad (9\text{-a})$$

$$V_y = -\frac{\partial \Phi}{\partial y} = -Awe^{k(z + b)}\sin\alpha \cos\tau \qquad (9\text{-b})$$

$$V_z = -\frac{\partial \Phi}{\partial z} = Awe^{k(z + b)}\sin\tau \qquad (9\text{-c})$$

The water accelerations can be determined by taking the time derivatives of the velocities, and are:

$$\dot{V}_x = Aw^2e^{k(z + b)}\cos\alpha \sin\tau \qquad (10\text{-a})$$

$$\dot{V}_y = -Aw^2e^{k(z + b)}\sin\alpha \sin\tau \qquad (10\text{-b})$$

$$\dot{V}_z = -Aw^2e^{k(z + b)}\cos\tau \qquad (10\text{-c})$$

The relative velocities and accelerations can be obtained by calculating the water velocities and accelerations at the desired segment, and then subtracting the segment motions.

The forces acting on each segment can be resolved into six components:

X_{ci} is the force on the i-th column in the x-direction.
Y_{ci} is the force on the i-th column in the y-direction.
Z_{ci} is the force on the i-th column in the z-direction.

X_{hi} is the force on the i-th hull in the x-direction.
Y_{hi} is the force on the i-th hull in the y-direction.
Z_{hi} is the force on the i-th hull in the z-direction.

The hydrodynamic inertia for the motion of a cylinder along its longitudinal axis is negligible. The hydrodynamic inertia for the transverse motion of a slender vertical cylinder can be approximated by its displaced mass (ref. 1). The hydrodynamic inertia for the transverse motion of a horizontal cylinder below a free surface is a function of the oscillation frequency and submergence ratio (depth/radius) of the cylinder, and is plotted in figure 6 of reference 1.

The damping force is composed of wave damping and eddy damping. The wave damping is associated with progressive waves radiating outward from the body, and it is linear with respect to the relative velocity. The wave damping force due to the columns is negligible and a wave damping parameter for the hull is plotted in figure 7 of reference 1 as a function of oscillation frequency and submergence ratio. The wave damping coefficent is then expressed as:

$$C_w = \delta m_h w \tag{11}$$

The eddy drag is associated with the generation of turbulent eddies around the segments and is nearly proportional to relative velocity squared. The motion responses of the platform are small in the range outside of resonance, therefore this damping is small and can be neglected, however near resonance it must be included. The eddy drag normal to a cylinder in cross flow is:

$$D = c_d \tfrac{1}{2} \rho S |V| V \tag{12}$$

where: c_d = drag coefficient

ρ = water density

S = projected area of the cylinder

V = relative velocity

The absolute value of V is used in equation (12) to show that the force is always in the direction of the relative velocity. The value of c_d is determined experimentally, and an average value of 1.0 seems reasonable (ref. 1).

The relative velocity in equation (12) can be written in the form:

$$V = w\,X\cos(wt - \varepsilon) \tag{13}$$

where:

w = frequency of oscillation

X = amplitude of oscillation

ε = phase angle

Since the velocity is periodic, the eddy drag can be linearized using the describing function technique (ref. 5, 6), and written in the form: $D_v = C_v V$. The describing function approach assumes a periodic input (V) and further assumes that the only significant component of the output (drag) is the component at the input frequency.

The describing function can be written as the ratio of the fundamental drag component to the velocity amplitude.

$$C_v = \frac{D_1}{wX} \tag{14}$$

The fundamental drag component can be determined from the Fourier series expansion of equation (12). The resulting expression for the linearized eddy drag coefficient is:

$$C_v = \frac{8}{3\pi}\, c_d \tfrac{1}{2} \rho S w X \tag{15}$$

The linearized damping coefficients for the i-th segment can then be written as:

$$C_{xi} = \frac{4}{3\pi} c_d \rho d_c H w \chi_{cxi} \qquad (16\text{-a})$$

$$C_{yi} = \frac{4}{3\pi} c_d \rho d_c H w \chi_{cyi} \qquad (16\text{-b})$$

$$H_{yi} = \delta m_h w + \frac{4}{3\pi} c_d \rho d_h \tfrac{1}{2} L w \chi_{hyi} \qquad (16\text{-c})$$

$$H_{zi} = \delta m_h w + \frac{4}{3\pi} c_d \rho d_h \tfrac{1}{2} L w \chi_{hzi} \qquad (16\text{-d})$$

χ_{cxi} is the mean amplitude of displacement of column i along the x-direction. χ_{cyi}, χ_{hyi}, χ_{hzi} can be interpreted in a similar manner.

The details of the calculation of force components, as adapted from reference 1, are shown in appendix A, and are expressed as follows:

$$X_{ci} = -(m_c \ddot{\xi}_{ci} + C_{xi} \dot{\xi}_{ci}) + 3m_c w^2 A Q_o \cos\alpha \sin\tau_i$$
$$+ C_{xi} w A Q_o \cos\alpha \cos\tau_i \qquad (17\text{-a})$$

$$Y_{ci} = -(m_c \ddot{\eta}_{ci} + C_{yi} \dot{\eta}_{ci}) - 3m_c w^2 A Q_o \sin\alpha \sin\tau_i$$
$$-C_{yi} w A Q_o \sin\alpha \cos\tau_i \qquad (17\text{-b})$$

$$Z_{ci} = -\frac{\pi}{4} \rho g d_c^2 \zeta_i - (m_c w^2 Q_o - \frac{\pi}{4} \rho g d_c^2) A \cos\tau_i \qquad (17\text{-c})$$

$$X_{hi} = m_h w^2 A \mu \cos\alpha \sin\tau_i \qquad (17\text{-d})$$

$$Y_{hi} = -(\Delta m_h \ddot{\eta}_{hi} + H_{yi} \dot{\eta}_{hi}) - (m_h + 2\Delta m_h) w^2 A \mu \sin\alpha \sin\tau_i$$
$$- H_{yi} w A \mu \sin\alpha \cos\tau_i \qquad (17\text{-e})$$

$$Z_{hi} = -(\Delta m_h \ddot{\zeta}_i + H_{zi} \dot{\zeta}_i) - (m_h + 2\Delta m_h) w^2 A \mu \cos\tau_i$$
$$+ H_{zi} w A \mu \sin\tau_i \qquad (17\text{-f})$$

The equations of motion are determined by summing the forces and moments for each segment. The moments are determined by calculating the force per unit length, multiplying by the moment arm, and then integrating over the surface, being careful to include buoyancy effects in the moment calculations (ref. 1).

$$M\ddot{\xi} = \sum_{i=1}^{4}(X_{ci} + X_{hi}) \tag{18-a}$$

$$M\ddot{\eta} = \sum_{i=1}^{4}(Y_{ci} + Y_{hi}) \tag{18-b}$$

$$M\ddot{\zeta} = \sum_{i=1}^{4}(Z_{ci} + Z_{hi}) \tag{18-c}$$

$$I_x\ddot{\phi} = \sum_{i=1}^{4}\Big\{-\int_H\left(\frac{d}{dz}Y_{ci}\right)z\,dz + Y_{hi}z_h + (Z_{ci} + Z_{hi})y_{ci} + \left(m_{hi}z_h - \int_H \frac{\pi}{4}\rho d_{ci}^2 z\,dz\right)\phi\Big\} \tag{18-d}$$

$$I_y\ddot{\theta} = \sum_{i=1}^{4}\Big\{\int_H\left(\frac{d}{dz}X_{ci}\right)z\,dz - X_{ci}z_h - Z_{ci}x_{ci} - \int_{L_i}\left(\frac{d}{dx}Z_{hi}\right)x\,dx + \left(m_{hi}z_h - \int_H\frac{\pi}{4}\rho d_{ci}^2 z\,dz\right)\theta\Big\} \tag{18-e}$$

$$I_z\ddot{\psi} = \sum_{i=1}^{4}\Big\{-(X_{ci} + X_{hi})y_{ci} + Y_{ci}x_{ci} + \int_{L_i}\left(\frac{d}{dx}Y_{hi}\right)x\,dx\Big\} \tag{18-f}$$

When equations (17) are substituted into equations (18), the equations of motion become:

$$a_{11}\ddot{\xi} + a_{15}\ddot{\theta} + b_{11}\dot{\xi} + b_{15}\dot{\theta} = F_1 \tag{19-a}$$

$$a_{22}\ddot{\eta} + a_{24}\ddot{\phi} + b_{22}\dot{\eta} + b_{24}\dot{\phi} = F_2 \tag{19-b}$$

$$a_{33}\ddot{\zeta} + b_{33}\dot{\zeta} + c_{33}\zeta = F_3 \tag{19-c}$$

$$a_{24} \ddot{\eta} + a_{44} \ddot{\varphi} + b_{24} \dot{\eta} + b_{44} \dot{\varphi} + c_{44} \varphi = F_4 \quad (19\text{-d})$$

$$a_{15} \ddot{\xi} + a_{55} \ddot{\theta} + b_{15} \dot{\xi} + b_{55} \dot{\theta} + c_{55} \theta = F_5 \quad (19\text{-e})$$

$$a_{66} \ddot{\psi} + b_{66} \dot{\psi} = F_6 \quad (19\text{-f})$$

where the coefficients are defined in appendix B.

IV SOLUTION OF EQUATIONS OF MOTION

Since the control of heave, roll, and pitch are of primary interest, the motions of the SFSV in surge, sway, and yaw can be eliminated from equations (19), resulting in three uncoupled differential equations. As outlined in appendix B, these equations are of the form:

$$a_{33}\ddot{\zeta} + b_{33}\dot{\zeta} + c_{33}\zeta = f_{3s}\sin wt + f_{3c}\cos wt \quad (20\text{-a})$$

$$C_{13}\dddot{\theta} + C_{12}\ddot{\theta} + C_{11}\dot{\theta} + C_{10}\theta = J_{1s}\sin wt + J_{1c}\cos wt \quad (20\text{-b})$$

$$C_{23}\dddot{\phi} + C_{22}\ddot{\phi} + C_{21}\dot{\phi} + C_{20}\phi = J_{2s}\sin wt + J_{2c}\cos wt \quad (20\text{-c})$$

Equations (20) can be solved to determine the steady-state, uncontrolled motions. Details of the solution are given in appendix C. For a given frequency, the motions of the SFSV are:

$$\zeta = \zeta_o \cos(wt - \varepsilon_\zeta) \quad (21\text{-a})$$

$$\theta = \theta_o \cos(wt - \varepsilon_\theta) \quad (21\text{-b})$$

$$\phi = \phi_o \cos(wt - \varepsilon_\phi) \quad (21\text{-c})$$

ζ_o, θ_o, and ϕ_o are the heave, pitch, and roll amplitudes, and ε_ζ, ε_θ, and ε_ϕ are the corresponding phase angles relative to the wave.

V. OPTIMAL CONTROL

The desired control may be specified in many ways. One is simply a specification of the maximum allowable motions. This method has the advantage of simplicity in that an upper limit on the motion is specified, and the designer is faced with a specific criterion. However, the advantage of simplicity may become a disadvantage since the specification of a certain motion may be too expensive in terms of providing the required control. Therefore, rather than specifying an upper limit on motion, it is desirable to determine an optimal control.

One method of accomplishing this is to specify a performance index in terms of motion and the control, minimize the performance index with respect to the control, determine the optimal control, and then determine the resulting motions. One useful performance index is based on a cost function which is the sum of control cost and motion cost.

The cost of control can be expressed in terms of the capital cost required for installation of the pumping facilities, etc., the cost of operation, or some weighted combination of the two. Since the factors involved in the determination of cost of control can be estimated fairly accurately, an analytic expression for cost of control, as a function of control force, can be determined.

The cost of motion is less well defined and depends strongly on the mission. In determining this cost, the owner must decide how a particular motion will effect his operation (drilling, tracking, etc.). Then he must translate the effect into a cost indicator, expressed as a function of motion.

The total cost function can be expressed in the

form:

$$C = f_1 \text{ (motion)} + f_2 \text{ (control Action)}$$

often the two functions can be simplified, and the cost function becomes:

$$C = q_x X + p_x U \tag{22}$$

Where q_x is the cost of motion and p_x is the cost of control. U is the control force which is to be optimized. X is the motion of the SPSV, and can be a displacement from an equilibrium position, a velocity, an acceleration, or any combination.

A particular performance index is a quadratic form, integrated over one period.

$$P.I. = \int_0^T \left[(q_x X)^2 + (p_x U)^2 \right] dt \tag{23}$$

It is further assumed that the control force may be expressed analytically in the form:

$$U = U_x \cos (wt - \delta_x)$$

The motions are also of this form, $X = X_c \cos (wt - \varepsilon_{xc})$ where the subscript c refers to the fact that these motions are those occurring with the control forces applied.

The performance index can be integrated and expressed in terms of the motion amplitude, control force and costs. The result of integrating equation (23) is:

$$P.I. = \frac{\pi}{w} (q_x^2 X_c^2 + p_x^2 U_x^2) \tag{24}$$

To minimize the performance index, the derivative of

equation (24) with respect to t e control is equated to zero, giving:

$$\frac{2\pi}{w} q_x^2 X_c \frac{dX_c}{dU_x} + \frac{2\pi}{w} p_x^2 U_x = 0 \tag{25}$$

Equation (25) can be solved for the optimal control force, U_x, with the boundary condition that when the control force is zero, the amplitude of the motion is X_o (equations (21)). The optimal control force is then:

$$U_x = \frac{q_x}{p_x} (X_o^2 - X_c^2)^{1/2} \tag{26}$$

Since U_x is an additional force applied to the SPSV along with the wave force, it can be added to the right hand side of the equations of motion. Equations (20) can be solved to determine the controlled motions of the SPSV. The details of the calculation are presented in appendix D. The result is:

$$\Lambda^2 - K (1 - \Lambda^2)^{1/2} \cos(\gamma - \delta) - 1 = 0 \tag{27}$$

where:

Λ = amplitude factor = ratio of controlled to uncontrolled motion.

K = a factor which includes the ratio of cost of motion to cost of control, frequency, and physical characteristics of the SPSV.

δ = phase angle of the control force

γ = phase angle of the wave force.

Equation (27) gives the motions of the SPSV resulting from t e application of an optimal control force based on particular motion and control costs. This motion, in the form of an amplitude factor, is used to determine the power requirements.

VI POWER REQUIREMENTS

The power for the control system must provide for the pumping of enough water at a rate which will maintain the motions of the SPSV within the specified limits. The optimization of the control force has resulted in maximum allowable controlled motions in heave, pitch, and roll.

To remain within the heave specification, added inertia must be provided to compensate for the uncontrolled heave in excess of that specified. This amount of inertia can be expressed as:

$$m_{\zeta} = 4\rho \frac{\pi}{4} d_c^2 \left(\overline{\zeta}_o - \overline{\zeta}_c \right) \tag{28}$$

Where ρ is the water density, $\frac{\pi}{4} d_c^2$ is the cross-sectional area of one column, $\overline{\zeta}_o - \overline{\zeta}_c$ is the excessive heave which must be controlled, and the expression is multiplied by 4 to include all four columns of the SPSV. The bar over the symbol for motion indicates a time-varying, periodic function, while the absence of a bar denotes the magnitude, (e.g. $\overline{\zeta} = \zeta_o \cos(wt - \varepsilon_{\zeta})$).

To remain within the roll and pitch specifications, added moment must be provided in a similar fashion. The amount of roll moment to be added is:

$$M_{\phi} = 4\rho \frac{\pi}{4} d_c^2 y_c^2 \left(\overline{\phi}_o - \overline{\phi}_c \right) \tag{29}$$

The amount of pitch moment to be added is:

$$M_{\theta} = 4\rho \frac{\pi}{4} d_c^2 x_c^2 \left(\overline{\theta}_o - \overline{\theta}_c \right) \tag{30}$$

The required power is the product of the required mass flow rate, and the head against which the pumps operate. From equations (28), (29), and (30), the mass flow rate in

heave, roll, and pitch are:

$$\dot{m}_\zeta = \rho \pi d_c^2 \, (\dot{\bar{\zeta}}_o - \dot{\bar{\zeta}}_c) \tag{31}$$

$$\dot{m}_\varphi = \rho \pi d_c^2 \, y_c \, (\dot{\bar{\varphi}}_o - \dot{\bar{\varphi}}_c) \tag{32}$$

$$\dot{m}_\theta = \rho \pi d_c^2 \, x_c \, (\dot{\bar{\theta}}_o - \dot{\bar{\theta}}_c) \tag{33}$$

The head at an arbitrary point a distance z_o feet below the still water line outside of the hulls is:

$$H_o = z_o + Ae^{-kz_o} \cos \tau - \bar{\zeta}_c + x \, \bar{\theta}_c - y \, \bar{\varphi}_c \tag{34}$$

The second term in equation (34) includes the effect of the waves, and the last three terms include the motion of the SPSV.

The head inside of the tank is:

$$H_i = \frac{p_T}{\rho g} + z_i \tag{35}$$

z_i is the water height inside the ballast tank above the pump, and p_T is the air gauge pressure in the tank. The height of the water in the tank can be expressed in terms of the mass of water in the tank as:

$$z_i = \frac{m}{\rho A_T} \tag{36}$$

where A_T is the tank cross section area and is assumed not to vary with z_i. m is the mass of the water in the tank at any time, as determined from equations (28), (29), or (30).

The total pump head is the difference between the external and internal heads.

$$H_p = z_o + Ae^{-kz_o} \cos \tau - \bar{\zeta}_c + x \, \bar{\theta}_c - y \, \bar{\varphi}_c - \frac{p_T}{\rho g} - \frac{m}{\rho A_T} \tag{37}$$

In heave, water must be added (or removed) so as not to create pitch or roll moments. The water must act at a point $x = 0$, $y = y_c$ in the port hull, and $x = 0$, $y = -y_c$ in the starboard hull. Furthermore, the amount of water pumped is the same for both hulls. The power required is the product of equations (31) and (37). Details of the calculation are presented in appendix E.

In roll, the control moments are generated by adding water to one hull, and removing the same amount from the other. In order that no pitching moments are generated, the water must act at $x = 0$, $y = y_c$ in the port hull, and $x = 0$, $y = -y_c$ in the starboard hull. The power required is the product of equations (32) and (37), and is detailed in appendix E.

In pitch, the control moments are generated by adding water forward and removing it aft, or vice-versa. Although the water can act at any symmetrical points along the x-axis, it is assumed that these points are the hull-column intersections. An equal amount of water thus acts at each inter ection. The power required is the product of equations (33) and (37) and is detailed in appendix E.

After the calculations in appendix E are carried out, the power required to control the SPSV in heave, roll, or pitch is of the form:

$$P = P_o\left[(BD\cos(wt-\eta) + \tfrac{1}{2}BC\cos(2wt-\eta-\lambda) + \tfrac{1}{2}BC\cos(\lambda-\eta)\right] \qquad (38)$$

The coefficients are defined in appendix E. This expression shows that the total power requirements are composed of a constant component, a sinusoidal component of frequency w, and a second harmonic component of frequency 2w.

The power required to pump water out of the tank is positive. The negative power is delivered when the water re-enters the tanks, but it is assumed that this power is

not feasibly recoverable. As a result, the average power requirement over one period, determined from equation (38), is composed of only the positive contribution of the instantaneous power, and is:

$$P = P_o \left[\frac{BD}{\pi} + \frac{BC}{2\pi} + \tfrac{1}{2} BC \cos(\lambda - \eta) \right] \qquad (39)$$

The average power is used to evaluate the feasibility of active displacement control for a particular application. Although peak power gives an indication of the maximum pumping facilities required, and therefore capital investment, the average power gives an indication of the operating costs over the system's life. Although both peak and average power requirements are important considerations, only the average power requirements will be used for comparison in the next section.

VII. DISCUSSION AND CONCLUSIONS

To determine the feasibility of active displacement control for a typical submarine pontoon supported vessel, the power required to provide various amounts of control under specified conditions is calculated. The characteristics of the SPSV are adapted from reference 7 and are listed in table 1.

The first case considered is that arising from the desire to have essentially zero motions in a moderate sea state. A sea state 3, with a wave amplitude of 7 feet and a period of 8 seconds is representative of this condition. The uncontrolled heave amplitude is calculated from the solution to equation (20-a) and is 1 foot, with a zero phase angle. Since the motion is to be zero, the amplitude factor (Λ) is zero. The resultant average power requirement is 1930 HP from equation (39).

The next case arises from the desire to reduce the heave motion in heavy seas to a level where work can continue. A sea state 6, with a wave amplitude of 18 feet and a period of 14.5 seconds is representative of this condition. The uncontrolled heave amplitude is 9.6 feet. For purposes of this discussion, it is assumed that control costs and motion costs have been determined which result in an amplitude factor of 0.35 from equation (27). In reality, the control costs and motion costs would be explicitly determined, however this determination is beyond the scope of this thesis. With an amplitude factor of 0.35, the resulting controlled motion is 3.36 feet. The average power required to maintain this heave amplitude is calculated from equation (39) and is 7540 HP.

In the case of a severe storm where the wave period approaches the natural period of the SPSV, a resonant condition could exist. It might be desirable to maintain the heave motion within a range where limited work could be performed. To provide a comparison to the previous example, a storm condition with wave amplitude of 18 feet and a period of

18.6 seconds, the natural period of heave, was considered. The resulting uncontrolled motion is 24.2 feet. Again for comparison with the previous example an amplitude factor is taken as 0.35, which gives a controlled heave amplitude of 8.5 feet. The average power required to provide this control is 28,000 HP.

If it is not feasible to provide small motions in the resonant condition, it might still be desirable to limit the heave amplitude to some maximum value. One obvious value is the wave amplitude. If the SPSV's heave amplitude, in the case previously considered, were limited to the wave amplitude, 18 feet, the corresponding amplitude factor is 0.744. The average power required to maintain this maximum heave is calculated from equation (39) and is 9700 HP.

Table 2 summarizes the preceding examples and also shows the complete power equations, equation (38).

The preceding calculations were based on the ballast tanks being vented to the atmosphere. The power equations in table 2 show that a significant portion of the average power arises from the first term, or the component with frequency w. If the amplitude of this component could be significantly reduced, the average power requirement could be similarly reduced. Appendix E shows that the coefficient BD depends among other things, upon the air pressure inside the tank. It was originally derived from a consideration of the difference in head across the ballast pump. If the tank were maintained at a constant pressure of 14.4 psi, then D = 0 and the power requirements are significantly reduced.

The natural period of the SPSV depends strongly on the column diameters. For example, if the diameters were reduced from 25 to 20 feet, the natural period of heave is increased from 18.6 to 23 seconds. Reduction of the waterplane area by reduction of the column diameters is thus a means of increasing the natural period to avoid a resonant heave condition. However,

changes in the column diameters are limited by other considerations. The primary limitation is the "spring constant", or tons per inch immersion (TPI) of the SPSV. Too great a reduction in the diameters may well result in unacceptable vertical displacements caused by addition or removal of weight. This also holds true for list and trim. One possible compromise which would permit small column diameters at normal operating conditions, yet would provide increased restoring forces at large drafts, is a column design where the diameter increases with submergence.

To maintain buoyancy with smaller columns, the hulls must be enlarged to provide the necessary volume. Structural considerations may also limit the minimum column diameters.

If it is not possible to increase the natural period, active displacement control might be considered. From table 2 it can be seen that the average power required to decrease heave in a resonant condition is extremely large if the desired amplitude reduction is large. However, if it is satisfactory only that the SPSV not exceed the wave amplitude, the power requirements are significantly decreased. Even in this situation, the power requirement is high, therefore it is probably more economical to design the SPSV with a high enough natural period so that resonant operation is avoided.

The use of active displacement control to extend the use of the SPSV into higher sea conditions appears practicable, although the power requirements are still fairly high. The greatest potential for active displacement control of the SPSV is to maintain very small motions in moderate sea conditions. The cost of such a system in terms of average power requirements is small, and although there are practical difficulties in the actual design, the concept should be pursued.

TABLE I

Characteristics of the SPSV

Platform length	185 ft
Platform breadth	185 ft
Hull length	240 ft
Hull diameter	25 ft
Column diameter	25 ft
Draft	45 ft
Clearance between water and platform	44 ft
Propulsive power	6000 HP
Natural periods of motion	
Heave	18.6 sec
Pitch	42.5 sec
Roll	35.0 sec

TABLE 2

Summary of Power Requirements

	I	II	III	IV
A	7	18	18	18
T	8	14.5	18.6	18.6
ζ_o	1	9.6	24.2	24.2
Λ	0	0.35	0.35	0.744
ζ_c	0	3.36	8.5	18
P (p_T= 0)	1930	7540	28000	9700
P (p_T= 14.4)	68.3	1150	15400	4880

Condition I is the state 3 sea.

Condition II is the state 6 sea.

Conditions III and IV are the resonant conditions.

The full power equations are:

$$P_I = 180 \left[32.5 \cos(wt - 270) + 1.19 \cos(2wt - 270) \right]$$

$$P_{II} = 9120 \left[2.2 \cos(wt - 80.5) + .123 \cos(2wt - 123.5) + 0.0865 \right]$$

$$P_{III} = 45200 \left[0.87 \cos(wt - 140) + 0.315 \cos(2wt - 240) + 0.241 \right]$$

$$P_{IV} = 45200 \left[0.344 \cos(wt - 140) + 0.108 \cos(2wt - 233) + 0.0737 \right]$$

APPENDIX A

DETAILS OF FORCE DERIVATION

The wave force on a column may be obtained from equation (5). On an element dz_i , the wave force in the y-direction is

$$- \frac{(m_c + \Delta m_c)}{m_c} \frac{\partial p}{\partial y} \frac{\pi d_c^2}{4} dz_i$$

The total wave force in the y-direction is:

$$- \frac{1}{H} \int_{-(H+b)}^{-b} (m_c + \Delta m_c) w^2 A e^{k(z_i + b)} \sin\alpha \sin\tau_i \, dz_i \qquad \text{(A-1)}$$

The force due to the hydrodynamic inertia is:

$$\frac{\Delta m_c}{H} \int_{-(H+b)}^{-b} (\text{relative acceleration in the y-direction}) \, dz_i \qquad \text{(A-2)}$$

The force due to damping is:

$$\frac{C_{yi}}{H} \int_{-(H+b)}^{-b} (\text{relative velocity in the y-direction}) \, dz_i \qquad \text{(A-3)}$$

Substituting the expressions for relative acceleration and velocity into equations (A-2) and (A-3), integrating and adding, the total force on a column in the y-direction becomes:

$$Y_{ci} = -\left[\Delta m_{ci} \ddot{\eta}_{ci} + C_{yi} \dot{\eta}_{ci}\right] - (m_{ci} + 2\Delta m_{ci}) w^2 A Q_o \sin\alpha \sin\tau_i$$

$$- C_{yi} w A Q_o \sin\alpha \cos\tau_i \qquad \text{(A-4)}$$

where $\eta_{ci} = \eta + c_{ci}\psi - z_c\phi$

The force on a column in the x-direction is derived in

a similar manner, and the result is:

$$X_{ci} = -\left[\Delta m_{ci}\ddot{\xi}_{ci} + C_{xi}\dot{\xi}_{ci}\right] + (m_{ci} + 2\Delta m_{ci})w^2AQ_o\cos\alpha\sin\tau_i$$

$$+ C_{xi}wAQ_o\cos\alpha\cos\tau_i \qquad \text{(A-5)}$$

where $\xi_{ci} = \xi + z_c\theta - y_{ci}\psi$

The derivation of the force on a hull in the y-direction is similar to that for a column. The resulting force is:

$$Y_{hi} = -\left[\Delta m_{hi}\ddot{\eta}_{hi} + H_{yi}\dot{\eta}_{hi}\right] - (m_{hi} + 2\Delta m_{hi})w^2A\mu\sin\alpha\sin\tau_i$$

$$- H_{yi}wA\mu\sin\alpha\cos\tau_i \qquad \text{(A-6)}$$

where $\eta_{hi} = \eta + x_{ci}\psi + z_h\phi$

In the x-direction, the drag and hydrodynamic inertia of the hull are negligable compared to that in the y-direction. Consequently, the wave force is the only significant force and is:

$$X_{hi} = m_{hi}w^2A\mu\cos\alpha\sin\tau_i \qquad \text{(A-7)}$$

The derivation of the force on a column in the z-direction is similar to that on a hull in the x-direction with the addition of a restoring force, and is:

$$Z_{ci} = -\tfrac{1}{4}\rho\pi g d_{ci}^2\zeta_i - (m_c w^2 Q_o - \tfrac{1}{4}\pi\rho g d_{ci}^2)A\cos\tau_i \qquad \text{(A-8)}$$

The derivation of the force on a hull in the z-direction is similar to that on a column in the x- and y-directions, and is:

$$Z_{hi} = -\left[\Delta m_{hi}\ddot{\zeta}_i + H_{zi}\dot{\zeta}_i\right] - (m_h + 2\Delta m_h)w^2 A\mu\cos\tau_i$$

$$+ H_{zi} wA\mu\sin\tau_i \qquad \text{(A-9)}$$

After some simplification, equations (A-4) through (A-9) are reproduced as equations (17).

APPENDIX B

DETAILS OF EQUATIONS OF MOTION

When the force compoments, equations (17), are substituted into equations (18), the equations of motions result in the following form: (see also reference 1)

$$\left[a_{jk}\right]\left[\ddot{x}_k\right] + \left[b_{jk}\right]\left[\dot{x}_k\right] + \left[c_{jk}\right]\left[x_k\right] = \left[F_j\right] \tag{B-1}$$

where:

$$\left[x_k\right] = \begin{bmatrix} \xi \\ \eta \\ \zeta \\ \phi \\ \theta \\ \psi \end{bmatrix} \quad \text{and} \quad \begin{matrix} \left[a_{jk}\right] = \left[a_{kj}\right] \\ \left[b_{jk}\right] = \left[b_{kj}\right] \\ \left[c_{jk}\right] = \left[c_{kj}\right] \end{matrix}$$

The non-zero coefficients are:

$$a_{11} = M + 4m_c$$

$$a_{15} = 4m_c z_1$$

$$a_{22} = M + 4m_c + 4\Delta m_h$$

$$a_{24} = -4m_c z_1 + 4\Delta m_h z_h$$

$$a_{33} = M + 4\Delta m_h$$

$$a_{44} = I_x + 4m_c z_2^2 + 4\Delta m_h z_h^2$$

$$a_{55} = I_y + 4m_c z_2^2 + 4\Delta m_h x_2^2$$

$$a_{66} = I_z + 4m_c(x_c^2 + y_c^2) + 4\Delta m_h x_2^2$$

$$b_{11} = 4C_x$$

$$b_{15} = 4z_1 C_x$$

$$b_{22} = C_x + H_y$$

$$b_{24} = -4z_1 C_y + 4z_h H_y$$

$$b_{33} = 4H_z$$
$$b_{44} = 4(C_y z_2^2 + H_y z_h^2)$$
$$b_{55} = 4(C_x z_2^2 + H_z x_2^2)$$
$$b_{66} = 4(y_c^2 C_x + x_c^2 C_y + x_2^2 H_y)$$

$$c_{33} = \pi \rho g d_c^2$$
$$c_{44} = \pi \rho g d_c^2 y_c^2 + 4m_c g z_1 - 4m_h g z_h$$
$$c_{55} = \pi \rho g d_c^2 x_c^2 + 4m_c g z_1 - 4m_h g z_h$$

$$F_1 = 4wAS \cos\alpha \left\{(-3Q_o m_c w - \mu m_h w) \sin wt + Q_o C_x \cos wt\right\}$$
$$F_2 = 4wAS \sin\alpha \left\{w\left[3Q_o m_c - \mu(m_h + \Delta m_h)\right] \sin wt - (Q_o C_y + \mu H_y) \cos wt\right\}$$
$$F_3 = -4wAS \left\{H_z \mu \sin wt + \left[m_c w Q_o - \frac{\pi}{4} \rho g \frac{d_c^2}{w} + (m_h + 2\Delta m_h) w \mu\right] \cos wt\right\}$$
$$F_4 = 4wAS \sin\alpha \left\{\left[-3m_c w Q_1 + (m_c w^2 Q_o - \frac{\pi}{4}\rho g d_c^2) \frac{y_c \tan(k y_c \sin\alpha)}{w \sin\alpha} + (m_h + 2\Delta m_h) w \mu z_h\right] \sin wt + (Q_1 C_y - z_h \mu H_y) \cos wt\right\}$$
$$F_5 = 4wAS \left\{\left[-3m_c w Q_1 \cos\alpha + (m_c w^2 Q_o - \frac{\pi}{4}\rho g d_c^2) x_c \frac{\tan(k y_c \sin\alpha)}{w} + m_h w \mu z_h \cos\alpha + \mu_e\left(\left[m_h + 2\Delta m_h\right] K_s + H_z K_c\right)\right] \sin wt + Q_1 C_x \cos\alpha + \mu_e\left(\left[m_h + 2\Delta m_h\right] K_c - H_z K_s\right)\right] \cos wt\right\}$$
$$F_6 = -4wAS \left\{\left[3m_c w Q_o \cos\alpha \; y_c + Q_o C_y \sin\alpha \; x_c \tan(k x_c \cos\alpha) - Q_o C_x \cos\alpha \; y_c \tan(k y_c \sin\alpha) - \mu_e\left(\left[m_h + 2\Delta m_h\right] \sin\alpha \, K_c - H_y \sin\alpha \, K_s\right)\right] \sin wt + \left[3m_c w Q_o \sin\alpha \, x_c \tan(k x_c \cos\alpha) + \mu_e\left(\left[m_h + 2\Delta m_h\right] \sin\alpha \, K_s + H_y \sin\alpha \, K_c\right)\right] \cos wt\right\}$$

Written out, the equations become:

$$a_{11} \ddot{\xi} + a_{15} \ddot{\theta} + b_{11} \dot{\xi} + b_{15} \dot{\theta} = F_1 \qquad \text{(B-2)}$$

$$a_{22} \ddot{\eta} + a_{24} \ddot{\phi} + b_{22} \dot{\eta} + b_{24} \dot{\phi} = F_2 \qquad \text{(B-3)}$$

$$a_{33} \ddot{\zeta} + b_{33} \dot{\zeta} + c_{33} \zeta = F_3 \qquad \text{(B-4)}$$

$$a_{24} \ddot{\eta} + a_{44} \ddot{\phi} + b_{24} \dot{\eta} + b_{44} \dot{\phi} + c_{44} \phi = F_4 \qquad \text{(B-5)}$$

$$a_{15} \ddot{\xi} + a_{55} \ddot{\theta} + b_{15} \dot{\xi} + b_{55} \dot{\theta} + c_{55} \theta = F_5 \qquad \text{(B-6)}$$

$$a_{66} \ddot{\psi} + b_{66} \dot{\psi} = F_6 \qquad \text{(B-7)}$$

Since heave, pitch, and roll are of primary significance, surge can be eliminated by combining equations (B-2) and (B-6), and sway can be eliminated by combining equations (B-3) and (B-5). Using the notation $D = \frac{d}{dt}$, equations (B-2) and (B-6) become:

$$D(a_{11}D + b_{11})\xi + D(a_{15}D + b_{15})\theta = F_1 \qquad \text{(B-8)}$$

$$D(a_{15}D + b_{15})\xi + (a_{55}D^2 + b_{55}D + c_{55})\theta = F_5 \qquad \text{(B-9)}$$

Solving simultaneously for θ :

$$\left[(a_{11}D + b_{11})(a_{55}D^2 + b_{55}D + c_{55}) - D(a_{15}D + b_{15})^2\right]\theta =$$

$$(a_{11}D + b_{11})F_5 - (a_{15}D + b_{15})F_1 \qquad \text{(B-10)}$$

Performing the required operations, and noting that:

$$F_i = f_{is}\sin wt + f_{ic}\cos wt$$

where the coefficients are given above, equation (B-10) becomes:

$$C_{13} \dddot{\theta} + C_{12} \ddot{\theta} + C_{11} \dot{\theta} + C_{10} \theta = J_{1s}\sin wt + J_{1c}\cos wt \qquad \text{(B-11)}$$

where:

$$C_{13} = a_{11}a_{55} - a_{15}^2$$

$$C_{12} = a_{11}b_{55} + b_{11}a_{55} - 2a_{15}b_{15}$$

$$C_{11} = a_{11}c_{55} + b_{11}b_{55} - b_{15}^2$$

$$C_{10} = b_{11}c_{55}$$

$$J_{1s} = -a_{11}wf_{5c} + b_{11}f_{5s} + a_{15}wf_{1c} - b_{15}f_{1s}$$

$$J_{1c} = a_{11}wf_{5s} + b_{11}f_{5c} - a_{15}wf_{1s} - b_{15}f_{1c}$$

Noting that equations (B-3) and (B-5) are similar to equations (B-2) and (B-6), the differential equation for roll can be written by inspection:

$$C_{23}\dddot{\varphi} + C_{22}\ddot{\varphi} + C_{21}\dot{\varphi} + C_{20}\varphi = J_{2s}\sin wt + J_{2c}\cos wt \qquad \text{(B-12)}$$

where:

$$C_{23} = a_{22}a_{44} - a_{24}^2$$

$$C_{22} = a_{22}b_{44} + b_{22}a_{44} - 2a_{24}b_{24}$$

$$C_{21} = a_{22}c_{44} + b_{22}b_{44} - b_{24}^2$$

$$C_{20} = b_{22}c_{44}$$

$$J_{2s} = -a_{22}wf_{4c} + b_{22}f_{4s} + a_{24}wf_{2c} - b_{24}f_{2s}$$

$$J_{2c} = a_{22}wf_{4s} + b_{22}f_{4c} - a_{24}wf_{2s} - b_{24}f_{2c}$$

Summarizing, the equations of motion for the SPSV in heave, pitch, and roll are:

$$a_{33}\ddot{\zeta} + b_{33}\dot{\zeta} + c_{33}\zeta = f_{3s}\sin wt + f_{3c}\cos wt \qquad \text{(B-13)}$$

$$C_{13}\dddot{\theta} + C_{12}\ddot{\theta} + C_{11}\dot{\theta} + C_{10}\theta = J_{1s}\sin wt + J_{1c}\cos wt \qquad \text{(B-14)}$$

$$C_{23}\dddot{\varphi} + C_{22}\ddot{\varphi} + C_{21}\dot{\varphi} + C_{20}\varphi = J_{2s}\sin wt + J_{2c}\cos wt \qquad \text{(B-15)}$$

APPENDIX C

DETAILS OF SOLUTION OF EQUATIONS OF MOTION

The equations of motion in heave, pitch, and roll, equations (B-13), (B-14), and (B-15), can be solved by assuming solutions of the form:

$$\zeta = \zeta_o \cos(wt - \varepsilon_\zeta) \qquad \text{(C-1)}$$

$$\theta = \theta_o \cos(wt - \varepsilon_\theta) \qquad \text{(C-2)}$$

$$\phi = \phi_o \cos(wt - \varepsilon_\phi) \qquad \text{(C-3)}$$

Inserting these relationships into the equations of motion, equations (20), results in expressions which determine the magnitudes and phase angles of the SPSV motions.

Substituting equation (C-1) into (20-a):

$$\zeta_o\left[(-a_{33}w^2 + c_{33}) \cos(wt - \varepsilon_\zeta) + (-b_{33}w)\sin(wt - \varepsilon_\zeta)\right] =$$

$$f_{3s}\sin wt + f_{3c}\cos wt \qquad \text{(C-4)}$$

Combining sine and cosine terms on either side of equation (C-4) results in:

$$\zeta_o R_\zeta \cos(wt - \beta_\zeta) = F_\zeta \cos(wt - \gamma_\zeta) \qquad \text{(C-5)}$$

where:

$$R_\zeta = \left[(-a_{33}w^2 + c_{33})^2 + (-b_{33}w)^2\right]^{1/2}$$

$$\beta_\zeta = \varepsilon_\zeta + \tan^{-1}\left[\frac{-b_{33}w}{-a_{33}w^2 + c_{33}}\right]$$

$$F_\zeta = (f_{3s}^2 + f_{3c}^2)^{1/2}$$

$$\gamma_\zeta = \tan^{-1}\left[\frac{f_{3s}}{f_{3c}}\right]$$

For equation (C-2) to be true for arbitrary values of time:

$$\zeta_o = \frac{F_\zeta}{R_\zeta} \tag{C-6a}$$

and $\beta_\zeta = \gamma_\zeta$ or

$$\varepsilon_\zeta = \gamma_\zeta - \tan^{-1}\left[\frac{-b_{33}w}{-a_{33}w^2 + c_{33}}\right] \tag{C-6b}$$

Equations (C-2) and (C-3) are treated in a similar fashion, the presence of a third derivative adding one additional term. Substituting equations (C-2) and (C-3) into (B-14) and (B-15), simplifying, combining sine and cosine terms, and solving for the amplitudes and phase angles, the resulting expressions for pitch and roll become:

$$\theta_o = \frac{F_\theta}{R_\theta} \tag{C-7a}$$

$$\varepsilon_\theta = \gamma_\theta - \tan^{-1}\left[\frac{C_{13}w^3 - C_{11}w}{-C_{12}w^2 + C_{10}}\right] \tag{C-7b}$$

where:

$$R_\theta = \left[(-C_{12}w^2 + C_{10})^2 + (C_{13}w^3 - C_{11}w)^2\right]^{1/2}$$

$$F_\theta = (J_{1s}^2 + J_{1c}^2)^{1/2}$$

$$\gamma_\theta = \tan^{-1}\left[\frac{J_{1s}}{J_{1c}}\right]$$

and for roll:

$$\phi_o = \frac{F_\phi}{R_\phi} \tag{C-8a}$$

$$\varepsilon_\phi = \gamma_\phi - \tan^{-1}\left[\frac{C_{23}w^3 - C_{21}w}{-C_{22}w^2 + C_{20}}\right] \tag{C-8b}$$

where:

$$R_\phi = \left[(-C_{22}w^2 + C_{20})^2 + (C_{23}w^3 - C_{21}w)^2\right]^{1/2}$$

$$F_\phi = (J_{2s}^2 + J_{2c}^2)^{1/2}$$

$$\gamma_\phi = \tan^{-1}\left[\frac{J_{2s}}{J_{2c}}\right]$$

APPENDIX D

DETERMINATION OF CONTROLLED MOTIONS

To determine the controlled motions of the SPSV, the control force is added to the exciting force on the right hand side of the equations of motion, equations (20) as illustrated below for heave.

$$a_{33}\ddot{\zeta} + b_{33}\dot{\zeta} + c_{33}\zeta = f_{3s}\sin wt + f_{3c}\cos wt + U_{\zeta}\cos(wt - \delta_{\zeta}) \tag{D-1}$$

where: $U_{\zeta} = \dfrac{q_{\zeta}}{p_{\zeta}}\left(\zeta_o^2 - \zeta_c^2\right)^{1/2}$ from equation (26).

The controlled motion, $\zeta = \zeta_c \cos(wt - \varepsilon_{\zeta c})$ is substituted into equation (D-1) resulting in:

$$\zeta_c R_{\zeta}\cos(wt - \beta_{\zeta c}) = T_{\zeta}\cos(wt - \kappa_{\zeta}) \tag{D-2}$$

where:

$$R_{\zeta} = \left[(-a_{33}w^2 + c_{33})^2 + (-b_{33}w)^2\right]^{1/2}$$

$$\beta_{\zeta c} = \varepsilon_{\zeta c} + \tan^{-1}\left[\frac{-b_{33}w}{-a_{33}w^2 + c_{33}}\right]$$

$$T_{\zeta} = \left[F_{\zeta}^2 + U_{\zeta}^2 + 2F_{\zeta}U_{\zeta}\cos(\gamma_{\zeta} - \delta_{\zeta})\right]^{1/2}$$

$$\kappa_{\zeta} = \tan^{-1}\left[\frac{F_{\zeta}\sin\gamma_{\zeta} + U_{\zeta}\sin\delta_{\zeta}}{F_{\zeta}\cos\gamma_{\zeta} + U_{\zeta}\sin\delta_{\zeta}}\right]$$

and F_{ζ} and γ_{ζ} are defined in appendix C. Equation (D-2) results in:

$$\zeta_c = \frac{T_{\zeta}}{R_{\zeta}} \tag{D-3a}$$

and $$\varepsilon_{\zeta c} = \kappa_{\zeta} - \tan^{-1}\left[\frac{-b_{33}w}{-a_{33}w^2 + c_{33}}\right] \tag{D-3b}$$

Expanding equation (D-3a) and squaring:

$$\zeta_c^2 = \frac{T_\zeta^2}{R_\zeta^2} = \frac{F_\zeta^2 + U_\zeta^2 + 2F_\zeta U_\zeta \cos(\gamma_\zeta - \delta_\zeta)}{R_\zeta^2} \tag{D-4}$$

From equation (C-6a), $\zeta_o = \frac{F_\zeta}{R_\zeta}$, therefore

$$\zeta_c^2 = \zeta_o^2 + 2\zeta_o \frac{U_\zeta}{R_\zeta} \cos(\gamma_\zeta - \delta_\zeta) + \frac{U_\zeta^2}{R_\zeta^2} \tag{D-5}$$

Substituting U_ζ from equation (26) into equation (D-5),

$$\zeta_c^2 = \zeta_o^2 + 2\frac{\zeta_o}{R_\zeta}\frac{q_\zeta}{p_\zeta}(\zeta_o^2 - \zeta_c^2)^{1/2} \cos(\gamma_\zeta - \delta_\zeta) + \left(\frac{q_\zeta}{p_\zeta}\right)^2 \frac{(\zeta_o^2 - \zeta_c^2)}{R_\zeta^2} \tag{D-6}$$

Define an amplitude factor (Λ) as the ratio of the controlled motion to the uncontrolled motion:

$$\Lambda_\zeta \equiv \frac{\zeta_c}{\zeta_o} \tag{D-7}$$

When equation (D-7) is substituted into equation (D-6) and the expression simplified, the result is:

$$\Lambda_\zeta^2 - 2\frac{\dfrac{q_\zeta}{R_\zeta p_\zeta}}{1 + \left(\dfrac{q_\zeta}{R_\zeta p_\zeta}\right)^2}(1 - \Lambda_\zeta^2)^{1/2} \cos(\gamma_\zeta - \delta_\zeta) - 1 = 0 \tag{D-8}$$

The motions of pitch and roll are treated in a similar manner, with the subscript ζ replaced by θ or ϕ as applicable. The resulting equations are similar to equation (D-8). Without subscripts, the general equation can be

written:

$$\Lambda^2 - K\,(1 - \Lambda^2)^{1/2} \cos(\gamma - \delta) - 1 = 0 \qquad \text{(D-9)}$$

where: $K = 2\,\dfrac{\frac{q}{Rp}}{1 + \left(\frac{q}{Rp}\right)^2}$

Equation (D-9) is a general expression relating controlled motions of the SPSV, resulting from the optimal control force, to the phase angle of the control force, for given values of the control cost and motion cost.

APPENDIX E

DETAILS OF POWER CALCULATIONS

The power requirement in heave is the product of the heave mass flow rate, equation (31), and the head, equation (37). Since heave is the only motion considered, $\Theta_c = \phi_c = 0$. The mass flow rate for each hull is:

$$\dot{m}_\zeta = -\tfrac{1}{2}\rho\pi w d_c^2 \left(\zeta_o \sin(wt - \varepsilon_\zeta) - \zeta_c \sin(wt - \varepsilon_{\zeta c})\right) \qquad \text{(E-1)}$$

The head seen by the port hull is:

$$H_p = z_o - \frac{p_T}{\rho g} + Ae^{-kz_o}\cos(wt + ky_c \sin\alpha) - \zeta_c \cos(wt - \varepsilon_{\zeta c})$$
$$- \frac{\pi\, d_c^2}{2A_{T\zeta}}\left[\zeta_o \cos(wt - \varepsilon_\zeta) - \zeta_c \cos(wt - \varepsilon_{\zeta c})\right] \qquad \text{(E-2)}$$

The head seen by the starboard hull differs only in the sign of y_c. The power requirement is therefore:

$$P_\zeta = \dot{m}_\zeta H_p + \dot{m}_\zeta H_s = \dot{m}_\zeta (H_p + H_s) \qquad \text{(E-3)}$$

When the expressions for head and mass flow rate are substituted into equation (E-3), and the equation is expanded and simplified, the power equation becomes:

$$P_\zeta = P_{o\zeta}\left[B_\zeta D_\zeta \cos(wt - \eta_\zeta) + \frac{B_\zeta C_\zeta}{2}\cos(2wt - \eta_\zeta - \lambda_\zeta)\right.$$
$$\left. + \frac{B_\zeta C_\zeta}{2}\cos(\lambda_\zeta - \eta_\zeta)\right] \qquad \text{(E-4)}$$

where:

$$P_{o\zeta} = \rho\pi d_c^2 w\, \zeta_o^2$$

$$B_\zeta = \left[1 + \Lambda_\zeta^2 - 2\Lambda_\zeta \cos(\varepsilon_{\zeta c} - \varepsilon_\zeta)\right]^{1/2}$$

APPENDIX E

DETAILS OF POWER CALCULATIONS

The power requirement in heave is the product of the heave mass flow rate, equation (31), and the head, equation (37). Since heave is the only motion considered, $\theta_c = \phi_c = 0$. The mass flow rate for each hull is:

$$\dot{m}_\zeta = -\tfrac{1}{2}\rho\pi w d_c^2 \left(\zeta_o \sin(wt - \varepsilon_\zeta) - \zeta_c \sin(wt - \varepsilon_{\zeta c})\right) \qquad \text{(E-1)}$$

The head seen by the port hull is:

$$H_p = z_o - \frac{p_T}{\rho g} + Ae^{-kz_o}\cos(wt + ky_c \sin\alpha) - \zeta_c \cos(wt - \varepsilon_{\zeta c})$$
$$- \frac{\pi d_c^2}{2A_{T\zeta}} \left[\zeta_o \cos(wt - \varepsilon_\zeta) - \zeta_c \cos(wt - \varepsilon_{\zeta c})\right] \qquad \text{(E-2)}$$

The head seen by the starboard hull differs only in the sign of y_c. The power requirement is therefore:

$$P_\zeta = \dot{m}_\zeta H_p + \dot{m}_\zeta H_s = \dot{m}_\zeta (H_p + H_s) \qquad \text{(E-3)}$$

When the expressions for head and mass flow rate are substituted into equation (E-3), and the equation is expanded and simplified, the power equation becomes:

$$P_\zeta = P_{o\zeta}\left[B_\zeta D_\zeta \cos(wt - \eta_\zeta) + \frac{B_\zeta C_\zeta}{2}\cos(2wt - \eta_\zeta - \lambda_\zeta)\right.$$
$$\left. + \frac{B_\zeta C_\zeta}{2}\cos(\lambda_\zeta - \eta_\zeta)\right] \qquad \text{(E-4)}$$

where:

$$P_{o\zeta} = \rho\pi d_c^2 w \zeta_o^2$$

$$B_\zeta = \left[1 + \Lambda_\zeta^2 - 2\Lambda_\zeta \cos(\varepsilon_{\zeta c} - \varepsilon_\zeta)\right]^{1/2}$$

$$\eta_\zeta = \tan^{-1}\left[\frac{\Lambda_\zeta \cos\,\varepsilon_{\zeta c} - \cos\,\varepsilon_\zeta}{\Lambda_\zeta \sin\,\varepsilon_{\zeta c} - \sin\,\varepsilon_\zeta}\right]$$

$$D_\zeta = \frac{1}{\zeta_o}\left(z_o - \frac{p_T}{\rho g}\right)$$

$$C_\zeta = \left\{\left[\frac{A}{\zeta_o} e^{-kz_o}\cos(ky_c \sin\alpha) + \left(\frac{\pi d_c^2}{2A_{T\zeta}} - 1\right)\Lambda_\zeta \cos\varepsilon_{\zeta c} - \frac{\pi d_c^2}{2A_{T\zeta}}\cos\varepsilon_\zeta\right]^2 + \left[\left(\frac{\pi d_c^2}{2A_{T\zeta}} - 1\right)\Lambda_\zeta \sin\varepsilon_{\zeta c} - \frac{\pi d_c^2}{2A_{T\zeta}}\sin\varepsilon_\zeta\right]^2\right\}^{1/2}$$

$$\lambda_\zeta = -\tan^{-1}\left[\frac{\left(\frac{\pi d_c^2}{2A_{T\zeta}} - 1\right)\Lambda_\zeta \sin\varepsilon_{\zeta c} - \frac{\pi d_c^2}{2A_{T\zeta}}\sin\varepsilon_\zeta}{\frac{A}{\zeta_o} e^{-kz_o}\cos(ky_c \sin\alpha) + \left(\frac{\pi d_c^2}{2A_{T\zeta}} - 1\right)\Lambda_\zeta \cos\varepsilon_{\zeta c} - \frac{\pi d_c^2}{2A_{T\zeta}}\cos\varepsilon_\zeta}\right]$$

The power required to control roll is determined from the product of equations (32) and (37). The mass flow rate for each hull is:

$$\dot{m}_\phi = -\tfrac{1}{2}\rho\pi w d_c^2 y_c\left[\phi_o \sin(wt - \varepsilon_\phi) - \phi_c \sin(wt - \varepsilon_{\phi c})\right] \qquad \text{(E-5)}$$

The head seen by the port hull is:

$$H_p = z_o - \frac{p_T}{\rho g} + Ae^{-kz_o}\cos(wt + ky_c \sin\alpha) + y_c\phi_c \cos(wt - \varepsilon_{\phi c}) - \frac{\pi d_c^2}{2A_{T\phi}} y_c\left[\phi_o \cos(wt - \varepsilon_\phi) - \phi_c \cos(wt - \varepsilon_{\phi c})\right] \qquad \text{(E-6)}$$

The head seen by the starboard hull differs only in the sign of y_c in the third term. The power requirement is therefore:

$$P_\phi = \dot{m}_\phi (H_p + H_s) \qquad \text{(E-7)}$$

Substituting the expressions for head and mass flow rate, the resulting power equation becomes:

$$P_\phi = P_{o\phi}\Big[B_\phi D_\phi \cos(wt - \eta_\phi) + \tfrac{1}{2}B_\phi C_\phi \cos(2wt - \eta_\phi - \lambda_\phi)$$

$$+ \tfrac{1}{2}B_\phi C_\phi \cos(\lambda_\phi - \eta_\phi)\Big] \qquad \text{(E-8)}$$

where:

$$P_{o\phi} = \rho \pi d_c^2 w (y_c \phi_o)^2$$

$$B = \left[1 + \Lambda_\phi^2 - 2\Lambda_\phi \cos(\varepsilon_{\phi c} - \varepsilon_\phi)\right]^{1/2}$$

$$\eta_\phi = \tan^{-1}\left[\frac{\Lambda_\phi \cos \varepsilon_{\phi c} - \cos \varepsilon_\phi}{\Lambda_\phi \sin \varepsilon_{\phi c} - \sin \varepsilon_\phi}\right]$$

$$D = \frac{1}{y_c \phi_o}\left(z_o - \frac{p_T}{\rho g}\right)$$

$$C = \left\{\left[\frac{A}{y_c \phi_o} e^{-kz_o}\cos(ky_c \sin\alpha) + \left(\frac{\pi d_c^2}{2A_{T\phi}} + 1\right)\Lambda_\phi \cos \varepsilon_{\phi c}\right.\right.$$

$$\left.\left. - \frac{\pi d_c^2}{2A_{T\phi}}\cos \varepsilon_\phi\right]^2 + \left[\left(\frac{\pi d_c^2}{2A_{T\phi}} + 1\right)\Lambda_\phi \sin \varepsilon_{\phi c} - \frac{\pi d_c^2}{2A_{T\phi}} \sin \varepsilon_\phi\right]^2\right\}^{1/2}$$

$$\lambda_\phi = -\tan^{-1}$$

$$\left[\frac{\left(\frac{\pi d_c^2}{2A_{T\phi}} + 1\right)\Lambda_\phi \sin \varepsilon_{\phi c} - \frac{\pi d_c^2}{2A_{T\phi}} \sin \varepsilon_\phi}{\frac{A}{y_c \phi_o} e^{-kz_o}\cos(ky_c \sin\alpha) + \left(\frac{\pi d_c^2}{2A_{T\phi}} + 1\right)\Lambda_\phi \cos \varepsilon_{\phi c} - \frac{\pi d_c^2}{2A_{T\phi}} \cos \varepsilon_\phi}\right]$$

The power requirements for pitch are determined in a similar manner, and the power equation is:

$$P_\theta = P_{o\theta}\Big[B_\theta D_\theta \cos(wt - \eta_\theta) + \tfrac{1}{2}B_\theta C_\theta \cos(2wt - \eta_\theta - \lambda_\theta)$$

$$+ \tfrac{1}{2}B_\theta C_\theta \cos(\lambda_\theta - \eta_\theta)\Big] \qquad \text{(E-9)}$$

where:

$$P_{o\theta} = \rho \pi d_c^2 w (x_c \theta_o)^2$$

$$B_\theta = \left[1 + \Lambda_\theta^2 - 2\Lambda_\theta \cos(\varepsilon_{\theta c} - \varepsilon_\theta)\right]^{1/2}$$

$$\eta_\theta = \tan^{-1}\left[\frac{\Lambda_\theta \cos \varepsilon_{\theta c} - \cos \varepsilon_\theta}{\Lambda_\theta \sin \varepsilon_{\theta c} - \sin \varepsilon_\theta}\right]$$

$$D_\theta = \frac{1}{x_c \theta_o} \left(z_o - \frac{p_T}{\rho g}\right)$$

$$C_\theta = \left\{\left[\frac{AS}{x_c \theta_o} e^{-kz_o} + \left(\frac{\pi d_c^2}{4A_{T\theta}} + 1\right) \Lambda_\theta \cos \varepsilon_{\theta c} - \frac{\pi d_c^2}{4A_{T\theta}} \cos \varepsilon_\theta\right]^2 + \left[\left(\frac{\pi d_c^2}{4A_{T\theta}} + 1\right)\Lambda_\theta \sin \varepsilon_{\theta c} - \frac{\pi d_c^2}{4A_{T\theta}} \sin \varepsilon_\theta\right]^2\right\}^{1/2}$$

$$\lambda_\theta = -\tan^{-1}\left[\frac{\left(\frac{\pi d_c^2}{4A_{T\theta}} + 1\right) \Lambda_\theta \sin \varepsilon_{\theta c} - \frac{\pi d_c^2}{4A_{T\theta}} \sin \varepsilon_\theta}{\frac{AS}{x_c \theta_o} e^{-kz_o} + \left(\frac{\pi d_c^2}{4A_{T\theta}} + 1\right) \Lambda_\theta \cos \varepsilon_{\theta c} - \frac{\pi d_c^2}{4A_{T\theta}} \cos \varepsilon_\theta}\right]$$

REFERENCES

1. Wang, S., "The motions and Forces of a Floating Platform in Waves", NSRDC Hydromechanics Laboratory, Tech. Note 98, March 1968.

2. Brooks, R. L., and Jasper, N.H., "Statistics on Wave Heights and Periods for the North Atlantic Ocean", DTMB report 1091, September 1957.

3. Comstock, J. P., Principles of Naval Architecture, SNAME, New York, N.Y., 1967.

4. Korvin-Kroukovsky, B. V., Theory of Seakeeping, SNAME, New York, N.Y., 1961.

5. Gibson, J. E., Nonlinear Automatic Control, McGraw-Hill, New York, N.Y., 1963.

6. Truxal, J. G., Automatic Feedback Control System Synthesis, McGraw Hill, New York, N.Y., 1955

7. Jayne, G. L., "Military Application of Stable Ocean Platforms", Fifth Annual Technical Symposium, Association of Senior Engineers, NSSC, 22 March 1968.